17216

16117

3 Hours / 100 Marks Seat No. | | | | | | | | |

Instructions – (1) All Questions are *Compulsory.*

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Assume suitable data, if necessary.

(6) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Attempt any <u>TEN</u> of the following: **20**

a) If $(a - 2bi) + (b - 3ai) = 5 + 2i$ Find a and b.

b) Express in the form $x + iy$, $\dfrac{(2+i)^2}{2+3i}$

where $x, y \in$ R and $i = \sqrt{-1}$

c) If $f(x) = x^4 - 2x + 7$ find $f(0) + f(2)$.

d) If $f(x) = 16^x + \log_2^x$ Find the value of $f(1/4)$

e) Evaluate $\lim\limits_{x \to 1} \left(\dfrac{x^3 - 1}{x - 1}\right)$

f) Evaluate $\lim\limits_{x \to 0} \left(\dfrac{3\sin x + 4x}{7x - 2\tan x}\right)$

g) Evaluate $\lim\limits_{x \to 0} \left(\dfrac{3^x - 2^x}{\sin \pi x}\right)$

h) If $y = e^{7x} . \cos 7x$ Find $\dfrac{dy}{dx}$.

Marks

i) If $y = \log(x.\sin 2x)$ Find $\frac{dy}{dx}$.

j) If $x = 3\sin 4\theta$, $y = 4\cos 3\theta$ Find $\frac{dy}{dx}$.

k) Show that there exist a root of the equation $x^3 - 5x - 11 = 0$ between 2 and 3.

l) Solve the following equations by using Jacobi's method (only first iteration)

$4x - y + z = 4$, $x + 6y + 2z = 9$, $-x - 2y + 5z = 2$

2. Attempt any FOUR of the following: **16**

a) Express $(1 + i)$ in polar form.

b) Simplify using De Moiver's theorem.

$$\frac{(\cos 2\theta + j\sin 2\theta)^{3/2}.(\cos\theta - j\sin\theta)^3}{(\cos 3\theta - j\sin 3\theta)^2.(\cos 5\theta - j\sin 5\theta)^{2/5}}$$

c) Use De Moiver's theorem to solve $x^3 - 1 = 0$

d) Separate into real and imaginary parts of cosh $(\alpha + i\beta)$.

e) If $f(x) = \frac{1}{1-x}$ show that $f\left[f\{f(x)\}\right] = x$.

f) If $f(x) = x^2 - 3x + 4$. Find x if $f(1-x) = f(2x+1)$.

3. Attempt any FOUR of the following: **16**

a) If $y = f(x) = \frac{2x-3}{3x-2}$ show that $x = f(y)$.

b) Evaluate: $\lim_{x \to 3} \frac{x^2 - x - 6}{x^3 - 3x^2 + x - 3}$

c) Evaluate: $\lim_{x \to 2} \left(\frac{x}{x-2} - \frac{4}{x^2 - 2x}\right)$

d) Evaluate: $\lim_{\theta \to 0} \frac{2\sin\theta - \sin 2\theta}{\theta^3}$

e) Evaluate: $\lim_{x \to 0} \frac{6^x - 3^x - 2^x + 1}{x^2}$

f) Evaluate: $f(x) = \log\left(\frac{1+x}{1-x}\right)$

Prove that: $f(a) + f(b) = f\left(\frac{a+b}{1+ab}\right)$

Marks

4. Attempt any FOUR of the following: 16

a) Using First principle of derivative find derivative of $f(x) = \cos x$.

b) If u and v are differentiable functions of x then prove that

$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

c) If $y = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ find $\frac{dy}{dx}$.

d) If $4x + 3y = \log(4x - 3y)$ find $\frac{dy}{dx}$.

e) If $x^3 . y^2 = (x+y)^5$ show that $\frac{dy}{dx} = y/x$.

f) If $x = a(2\theta - \sin 2\theta)$, $y = a(1 - \cos 2\theta)$

Find $\frac{dy}{dx}$ at $\theta = \pi/4$.

5. Attempt any FOUR of the following: 16

a) Evaluate $\lim_{x \to 0} \frac{(5^x - 1)\tan x}{\sqrt{x^2+16} - 4}$

b) Evaluate $\lim_{x \to 3} \frac{\log x - \log 3}{x - 3}$

c) Using Bisection method find the approximate root of the equation $x^3 - 5x + 1 = 0$ (three iterations only)

d) Find the root of $x^2 - x - 1 = 0$ by using Regula Falsi Method up to third approximation.

e) Find the positive root of $x^3 + x - 1 = 0$ by Newton-Raphson method upto three iterations only.

f) Use Newton-Raphson method to find $\sqrt[3]{20}$ correct to three decimal places. (third iteration)

Marks

6. Attempt any FOUR of the following: 16

a) If $y = (x + \sqrt{x^2 + 1})^m$ show that $(x^2 + 1)\frac{d^2 y}{dx^2} + x\frac{dy}{dx} - m^2 y = 0$

b) If $x = 2\cos t - \cos 2t,\ y = 2\sin t - \sin 2t$ Find $\frac{d^2 y}{dx^2}$ at $t = \pi/2$.

c) Solve the following equations by Gauss-elimination method.

$x + 2y + 3z = 14,\ 3x + 3y + 5z = 24$

$4x + 5y + 7z = 35.$

d) Solve the following equations by Jacobi's method (take three iterations)

$5x + 2y + z = 12,\ x + 4y + 2z = 15,\ x + 2y + 5z = 20$

e) Solve the equations by Gauss-seidal method upto two iterations.

$10x + 2y + z = 9,\ x + 10y - z = -22,\ -2x + 3y + 10z = 22.$

f) Solve the following equations by Jacobi's method (Take three iterations

$10x + y + z = 12$

$x + 10y + z = 12$

$x + y + 10z = 12$